

A New Terminal Sliding Mode Tracking Control for A Class of Nonminimum Phase Systems with Uncertain Dynamics

Zhihong Man, Weixiang Shen, and Xinghuo Yu

Abstract – In this paper, a new terminal sliding mode tracking control scheme is developed for a class of nonminimum phase systems with uncertainties. It is shown that, unlike conventional linear or terminal sliding mode controls, the Lyapunov stability theory in this paper is used to determine the upper and the lower bounds of the control signal and its derivative. A dynamic control signal can then be designed, subject to the bounded conditions, to drive the terminal sliding variable to converge to zero, and, on the terminal sliding mode surface, the tracking error is guaranteed to converge to zero in a finite time. A simulation example is presented in support of the proposed robust tracking control scheme.

Index Terms— Terminal sliding mode control; nonminimum phase; system uncertainties; Lyapunov stability.

I. INTRODUCTION

The tracking control of nonminimum phase systems has been widely investigated by many researchers in the last three decades [1]-[17]. It is pointed out in [1]-[4] that the perfect tracking or asymptotic tracking control of nonminimum systems with zeros on the right-half s-plane cannot be achieved by using simple feedback control laws because of the unstable internal dynamics. It is also noted from [5]-[6] that the exact output tracking input can be found using the stable inversion approach, however, the challenge of implementing such a control strategy is that the entire desired reference signal must be known. The research results in [7]-[12] show that some bounded error tracking control with small tracking error can be achieved for some desired trajectories of particular interests. In order to handle the uncertainties and disturbances in the tracking control of nonminimum phase systems, some adaptive control schemes have been developed in [13]-[17]. However, it seems that no results on the asymptotic tracking control of nonminimum phase systems with large uncertainties have been reported.

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In this paper, we will develop a new tracking control scheme for a class of non-minimum phase systems with uncertain dynamics, based on terminal sliding mode control technique [18]-[21]. It will be shown that, unlike the design of conventional sliding mode control systems where Lyapunov stability theory is used to determine the values of control signals to guarantee the closed-loop stability [22]-[25], Lyapunov stability theory in this paper is used to determine upper and lower bounds of control signal and its first-order derivative, a dynamic control law can then be constructed, subject to the bounded conditions, to drive the terminal sliding variable to converge to zero in a finite time, and the tracking error between the desired reference signal and the system output can then converge to zero in a finite time on the terminal sliding mode surface. A simulation example is given to show the finite time error convergence and the strong robustness property with respect to large parameter uncertainties of a closed-loop nonminimum phase system using the proposed new terminal sliding mode tracking control scheme.

The paper is organized as follows. In Section 2, the upper and lower bounds of the control signal and its first derivative are determined in the sense of Lyapunov stability, and a dynamics control signal is constructed, subject to the bounded conditions, to guarantee a finite-time convergence of the tracking error. In Section 3, a simulation example is presented to show the effectiveness of the proposed new terminal sliding mode control scheme for the tracking control of nonminimum phase systems.

II. MAIN RESULT

Consider a class of second-order nonminimum phase systems described by the following differential equation:

$$\ddot{x}(t) = f(x, \dot{x}) + u(t) - b_1 \dot{u}(t) \quad (1)$$

where $u(t)$ and $x(t)$ are the system input and output, respectively, $f(x, \dot{x})$ is an unknown linear or nonlinear function, satisfying the following bounded condition:

$$|f(x, \dot{x})| \leq f_0(x, \dot{x}) \quad (2)$$

b_1 is an unknown parameter which is upper and lower-bounded by

$$0 < b_{10} < b_1 < b_{11} \quad (3)$$

the positive function $f_0(x, \dot{x})$ and the positive constants b_{10} and b_{11} in (2) and (3) are known.

In this paper, we assume that the control input $u(t)$, the system output $x(t)$, and its derivative $\dot{x}(t)$ are measurable. Let $x_r(t)$ be the desired reference signal for the system output $x(t)$ to follow. The tracking error between the system output $x(t)$ and the desired reference signal $x_r(t)$ is then defined as:

$$e(t) = x(t) - x_r(t) \quad (4)$$

In order to use the terminal sliding mode control technique for designing the tracking controller, we define the following terminal sliding variable [18]-[21]:

$$s(t) = |\dot{e}(t)|^{q/p} \text{sign}(\dot{e}) + \beta e(t) \quad (5)$$

where β is a positive constant, q and p are positive odd integers satisfying:

$$2 > q/p > 1 \quad (6)$$

Differentiate the sliding variable $s(t)$ with respect to time t , we obtain:

$$\begin{aligned} \dot{s}(t) &= \frac{q}{p} \dot{e}(t)^{\frac{q}{p}-1} \ddot{e}(t) + \beta \dot{e}(t) \\ &= \frac{q}{p} \dot{e}(t)^{\frac{q}{p}-1} (f(x, \dot{x}) + u(t) - b_1 \dot{u}(t) - \ddot{x}_r(t)) + \beta \dot{e}(t) \end{aligned} \quad (7)$$

Theorem 2.1: Consider the nonminimum phase system in (1). If the control signal $u(t)$ and its first-order derivative $\dot{u}(t)$ satisfy the following bounded conditions:

$$\text{If } s > 0 \quad \dot{u}(t) \geq \frac{1}{b_{10}} (|\ddot{x}_r(t)| + f_0(x, \dot{x})) \quad (8a)$$

$$\text{and} \quad u(t) < -\frac{P}{q} \beta |\dot{e}(t)|^{2-\frac{q}{p}} - \eta \text{sign}(s) \quad (8b)$$

$$\text{If } s < 0 \quad \dot{u}(t) \leq -\frac{1}{b_{10}} (|\ddot{x}_r(t)| + f_0(x, \dot{x})) \quad (9a)$$

$$\text{and} \quad u(t) > \frac{P}{q} \beta |\dot{e}(t)|^{2-\frac{q}{p}} - \eta \text{sign}(s) \quad (9b)$$

where η is a positive constant, then the output tracking error $e(t)$ will converge to zero in a finite time.

Proof: Defining a Lyapunov function candidate:

$$V = \frac{1}{2} s^2(t) \quad (10)$$

and differentiating V with respect to time t , we have

$$\begin{aligned} \dot{V} &= s(t) \dot{s}(t), \\ &= s(t) \left(\frac{q}{p} \dot{e}(t)^{\frac{q}{p}-1} (f(x, \dot{x}) + u(t) - b_1 \dot{u}(t) - \ddot{x}_r(t)) + \beta \dot{e}(t) \right) \\ &= \frac{q}{p} s(t) \dot{e}(t)^{\frac{q}{p}-1} (f(x, \dot{x}) - \ddot{x}_r(t) - b_1 \dot{u}(t)) \\ &\quad + s(t) \left(\frac{q}{p} \dot{e}(t)^{\frac{q}{p}-1} u(t) + \beta \dot{e}(t) \right) \end{aligned} \quad (11)$$

(i) If $s > 0$, using (8a) in the first term of (11), we have

$$\frac{q}{p} s(t) \dot{e}(t)^{\frac{q}{p}-1} (f(x, \dot{x}) - \ddot{x}_r(t) - b_1 \dot{u}(t))$$

$$\begin{aligned} &\leq \frac{q}{p} s(t) \dot{e}(t)^{\frac{q}{p}-1} \left(f(x, \dot{x}) - \ddot{x}_r(t) - \frac{b_1}{b_{10}} (|\ddot{x}_r(t)| + f_0(x, \dot{x})) \right) \\ &\leq -\frac{q}{p} s(t) \dot{e}(t)^{\frac{q}{p}-1} \left(\frac{b_1}{b_{10}} f_0(x, \dot{x}) - f(x, \dot{x}) \right) \\ &\quad - \frac{q}{p} s(t) \dot{e}(t)^{\frac{q}{p}-1} \left(\frac{b_1}{b_{10}} |\ddot{x}_r(t)| - |\ddot{x}_r(t)| \right) \\ &\leq -\frac{q}{p} s(t) \dot{e}(t)^{\frac{q}{p}-1} \left(\frac{b_1}{b_{10}} f_0(x, \dot{x}) - f_0(x, \dot{x}) \right) \\ &\quad - \frac{q}{p} s(t) \dot{e}(t)^{\frac{q}{p}-1} \left(\frac{b_1}{b_{10}} |\ddot{x}_r(t)| - |\ddot{x}_r(t)| \right) \\ &= -\frac{q}{p} s(t) \dot{e}(t)^{\frac{q}{p}-1} \left(\frac{b_1}{b_{10}} - 1 \right) (|\ddot{x}_r(t)| + f_0(x, \dot{x})) < 0 \end{aligned} \quad (12a)$$

and using (8b) in the second term of (11) leads to

$$\begin{aligned} &s(t) \left(\frac{q}{p} \dot{e}(t)^{\frac{q}{p}-1} u(t) + \beta \dot{e}(t) \right) \\ &\leq s(t) \left(\frac{q}{p} \dot{e}(t)^{\frac{q}{p}-1} \left(-\frac{P}{q} \beta |\dot{e}(t)|^{2-\frac{q}{p}} - \eta \text{sign}(s) \right) + \beta \dot{e}(t) \right) \\ &= s(t) \left(-\beta |\dot{e}(t)| + \beta \dot{e}(t) - \frac{q}{p} \dot{e}(t)^{\frac{q}{p}-1} \eta \text{sign}(s) \right) \\ &\leq -\frac{q}{p} s(t) \dot{e}(t)^{\frac{q}{p}-1} \eta \text{sign}(s) \\ &= -\rho(t) |s(t)| < 0 \end{aligned} \quad (12b)$$

with

$$\rho(t) = \eta \frac{q}{p} \dot{e}(t)^{\frac{q}{p}-1} > 0 \quad (13)$$

(12a) and (12b) show that

$$\begin{aligned} \dot{V} &< -\frac{q}{p} s(t) \dot{e}(t)^{\frac{q}{p}-1} \left(\frac{b_1}{b_{10}} - 1 \right) (|\ddot{x}_r(t)| + f_0(x, \dot{x})) \\ &\quad - \rho(t) |s(t)| \leq -\rho(t) |s(t)| < 0 \quad \text{for } s > 0 \end{aligned} \quad (14)$$

(ii) If $s < 0$, using (9a) in the first term of (11), we have

$$\begin{aligned} &\frac{q}{p} s(t) \dot{e}(t)^{\frac{q}{p}-1} (f(x, \dot{x}) - \ddot{x}_r(t) - b_1 \dot{u}(t)) \\ &\leq \frac{q}{p} s(t) \dot{e}(t)^{\frac{q}{p}-1} \left(f(x, \dot{x}) - \ddot{x}_r(t) + \frac{b_1}{b_{10}} (|\ddot{x}_r(t)| + f_0(x, \dot{x})) \right) \\ &\leq \frac{q}{p} s(t) \dot{e}(t)^{\frac{q}{p}-1} \left(\frac{b_1}{b_{10}} f_0(x, \dot{x}) - |f(x, \dot{x})| \right) \\ &\quad + \frac{q}{p} s(t) \dot{e}(t)^{\frac{q}{p}-1} \left(\frac{b_1}{b_{10}} |\ddot{x}_r(t)| - |\ddot{x}_r(t)| \right) \\ &= \frac{q}{p} s(t) \dot{e}(t)^{\frac{q}{p}-1} \left(\frac{b_1}{b_{10}} - 1 \right) (|\ddot{x}_r(t)| + f_0(x, \dot{x})) < 0 \end{aligned} \quad (15a)$$

and using (9b) in the second term of (10) leads to

$$s(t) \left(\frac{q}{p} \dot{e}(t)^{\frac{q}{p}-1} u(t) + \beta \dot{e}(t) \right)$$

$$\begin{aligned} &\leq s(t) \left(\frac{q}{p} \dot{e}(t)^{\frac{q}{p}-1} \left(\frac{p}{q} \beta |\dot{e}(t)|^{2-\frac{q}{p}} - \eta \text{sign}(s) \right) + \beta \dot{e}(t) \right) \\ &= s(t) (\beta |\dot{e}(t)| + \beta \dot{e}(t)) - \eta \frac{q}{p} \dot{e}(t)^{\frac{q}{p}-1} |s(t)| \\ &\leq -\rho(t) |s(t)| < 0 \end{aligned} \quad (15b)$$

(15a) and (15b) mean that

$$\dot{V} < -\rho(t) |s(t)| < 0 \text{ for } s \neq 0 \quad (16)$$

Remark 2.1: Although the expression $\rho(t)$ in (13) contains the factor $\dot{e}(t)^{\frac{q}{p}-1}$, it will be seen from Figure 3 and Remark 3.1 in Section 3 that $|\dot{e}(t)|$ is greater than a positive constant before $s(t)$ converges to zero in a finite time. Then, $\rho(t) \geq \delta > 0$, where δ is a positive constant. Therefore, according to Lyapunov stability theory, the terminal sliding variable $s(t)$ will be driven to zero in a finite-time if the control signal $u(t)$ is designed such that the bounded conditions in (8a) ~ (9b) are satisfied, and the tracking error $e(t)$ will converge to zero in a finite time on the terminal sliding mode surface $s(t) = 0$.

Remark 2.2: It has been shown in Theorem 2.1 that, because the nonminimum phase system (1) has a zero on the right half s -plane, the control signal and its derivative must satisfy the bounded conditions in (8a) ~ (9b) in order to guarantee that the control signal can drive the terminal sliding variable $s(t)$ to converge to zero. In this paper, the control signal $u(t)$ is constructed as follows:

(a) For $s > 0$, the changing rate of $u(t)$ is chosen as:

$$\dot{u}(t) = \frac{1}{b_{10}} (|\ddot{x}_r(t)| + f_0(x, \dot{x})) \quad (17)$$

which satisfies the bounded condition in (8a), and the control input $u(t)$ is updated as follows:

$$u(t) = u_n + \int_{t_1}^t \frac{1}{b_{10}} (|\ddot{x}_r(\tau)| + f_0(x, \dot{x})) d\tau \quad (18)$$

where u_n is a negative number or function, which is chosen such that the bounded condition in (8b) is satisfied, that is,

$$u_n + \int_{t_1}^t \frac{1}{b_{10}} (|\ddot{x}_r(\tau)| + f_0(x, \dot{x})) d\tau < -\frac{p}{q} \beta |\dot{e}(t)|^{2-\frac{q}{p}} - \eta \text{sign}(s) \quad (19a)$$

$$\text{or } u_n < -\int_{t_1}^t \frac{1}{b_{10}} (|\ddot{x}_r(\tau)| + f_0(x, \dot{x})) d\tau - \frac{p}{q} \beta |\dot{e}(t)|^{2-\frac{q}{p}} - \eta \text{sign}(s) \quad (19b)$$

In fact, if the absolute value of u_n is large enough, the bounded condition in (19a) or (19b) can always be satisfied.

(b) For $s < 0$, the changing rate of $u(t)$ is chosen as:

$$\dot{u}(t) = -\frac{1}{b_{10}} (|\ddot{x}_r(t)| + f_0(x, \dot{x})) \quad (20)$$

which satisfies the bounded condition in (9a), and the control input $u(t)$ is updated by:

$$u(t) = u_p - \int_{t_1}^t \frac{1}{b_{10}} (|\ddot{x}_r(\tau)| + f_0(x, \dot{x})) d\tau \quad (21)$$

where u_p is a positive number or function, which is chosen such that the bounded condition in (9b) is satisfied, that is,

$$u_p > \int_{t_1}^t \frac{1}{b_{10}} (|\ddot{x}_r(\tau)| + f_0(x, \dot{x})) d\tau + \frac{p}{q} \beta |\dot{e}(t)|^{2-\frac{q}{p}} - \eta \text{sign}(s) \quad (22)$$

It is seen that, if value of u_p is large enough, the bounded condition in (22) can always be satisfied.

Remark 2.3: Please note that the upper limit t of the integrals in (18) and (21) cannot go to infinity, because the terminal sliding variable $s(t)$ is driven to zero in a finite time, and the upper limit t of the integrals in (18) and (21) must be less than or equal to the convergence time of the terminal sliding variable $s(t)$. Therefore the variable structure control signal $u(t)$ in (18) and (21) is upper and lower bounded.

Remark 2.4: It is noted that, in the design of conventional sliding mode control systems in [22]-[25], Lyapunov stability theory is used to directly determine the values of control signals in order to guarantee the stability and error convergence of closed-loop systems. However, considering the characteristics of nonminimum phase systems in this paper, we use Lyapunov stability theory to determine the bound information of the control signal and its changing rate, and the dynamic control signal can then be constructed, subject to the bounded constraints, such that the finite time error convergence of closed-loop system is achieved.

Remark 2.5: It is easy to see that, in order to satisfy the bounded conditions in (8a) ~ (9b) the dynamic control signal may not be unique, and many different control signals, satisfying the bound conditions in (8a) ~ (9b), may be designed to achieve the finite time error convergence.

III. A SIMULATION EXAMPLE

In this simulation section, we consider the following second-order nonminimum phase linear system:

$$\ddot{x}(t) = a_2 \dot{x}(t) + a_1 x(t) + u(t) - b_1 \dot{u}(t) \quad (23)$$

where $a_1 = -2$, $a_2 = -3$, $b_1 = 2$, and system initial values are $x(0) = 0.9$ and $\dot{x}(0) = 0$.

Now, in this simulation, we assume that the system parameters a_1 , a_2 , and b_1 are unknown, but the following bounded conditions are known:

$$|a_1| \leq 2.5, \quad |a_2| \leq 3.5, \quad \text{and } 1.5 < b_1 < 2.5 \quad (24)$$

Then the upper bound of $f(x(t), \dot{x}(t)) = -3\dot{x}(t) - 2x(t)$ is of the form:

$$f_0(x(t), \dot{x}(t)) = 3.5|\dot{x}(t)| + 2.5|x(t)| \quad (25)$$

The desired reference signal for the system out $x(t)$ to follow in this example is given by

$$x_r(t) = 2 \sin(t) \quad (26)$$

and the terminal sliding variable is defined as:

$$s(t) = |\dot{e}(t)|^{3/1} \text{sign}(\dot{e}(t)) + 4e(t) \quad (27)$$

where the tracking error $e(t) \triangleq x(t) - x_r(t)$.

Figure 1.1 ~ Figure 1.4 show the good performance of the output tracking, tracking error $e(t)$, terminal sliding variable $s(t)$, and control input $u(t)$, respectively, where two parameters u_n and u_p in (17) and (20) are chosen as:

$$u_n = -1.5, \text{ and } u_p = 1.5 \quad (28)$$

It is seen that the terminal sliding variable $s(t)$ converges to zero in 1.5 seconds and the tracking error converges to zero in 2 seconds.

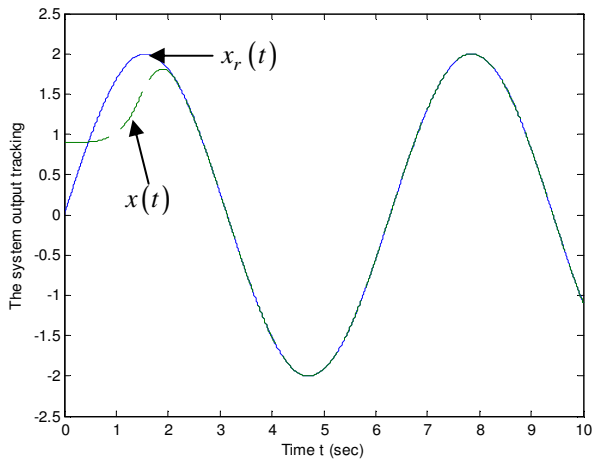


Fig. 1.1. The output tracking

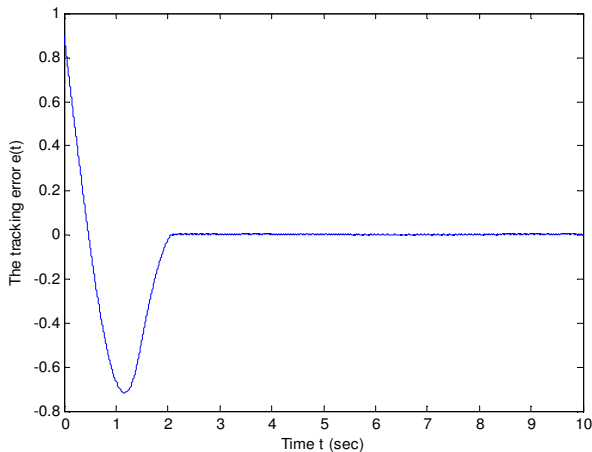


Fig. 1.2. The tracking error $e(t)$

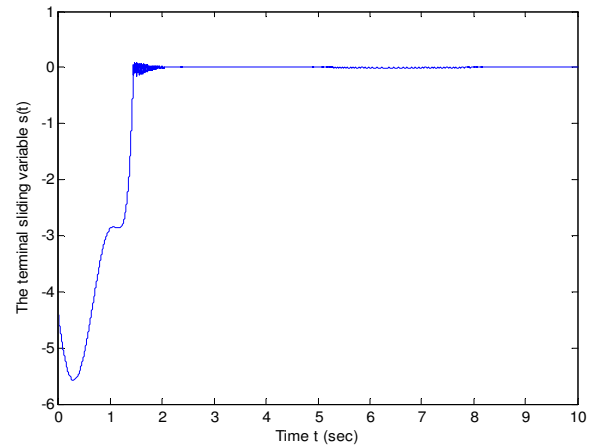


Fig. 1.3. Terminal sliding variable $s(t)$

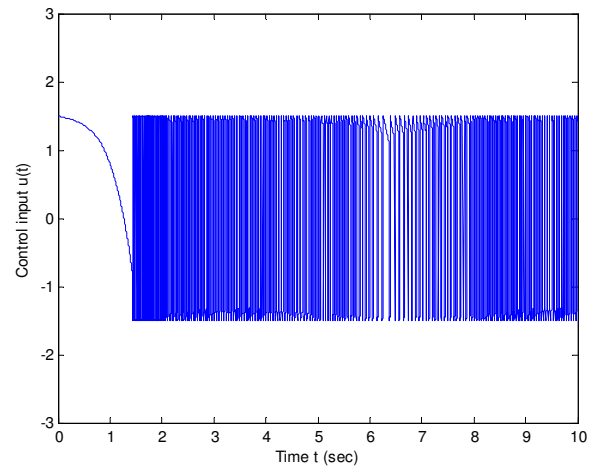


Fig. 1.4. Control input $u(t)$

Figure 2.1~Figure 2.4 show the simulation results where two parameters u_n and u_p in (17) and (20) are chosen as:

$$u_n = -4.5, \text{ and } u_p = 4.5 \quad (29)$$

It is seen that the terminal sliding variable $s(t)$ converges to zero in 0.3 seconds and the tracking error converges to zero in 0.9 seconds.

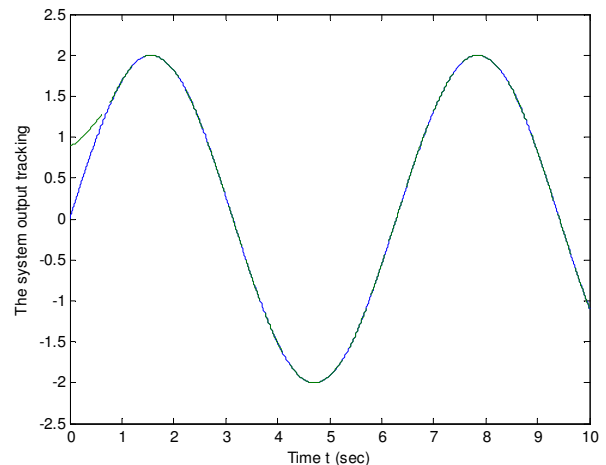


Fig. 2.1. The output tracking

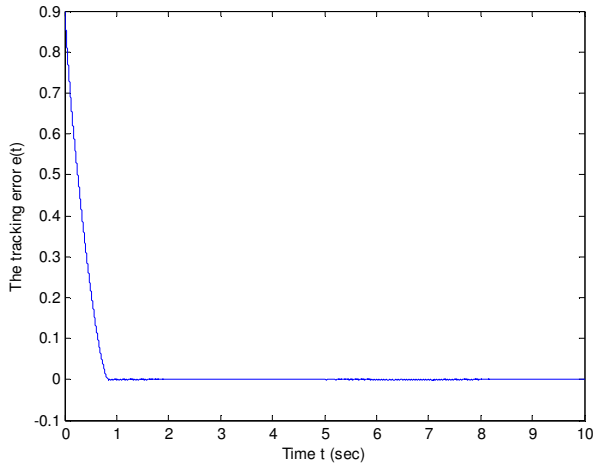


Fig. 2.2. The tracking error $e(t)$

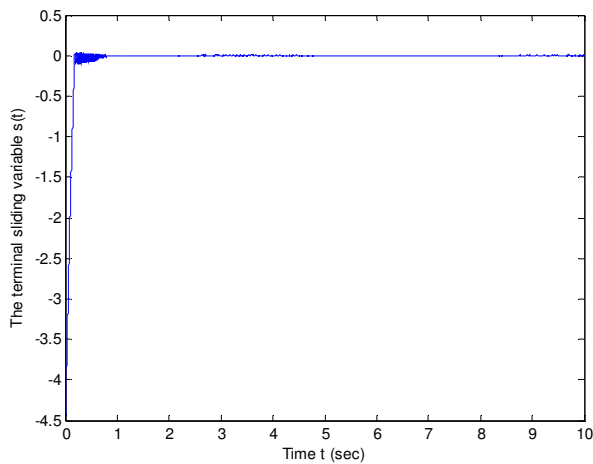


Fig. 2.3. Terminal sliding variable $s(t)$

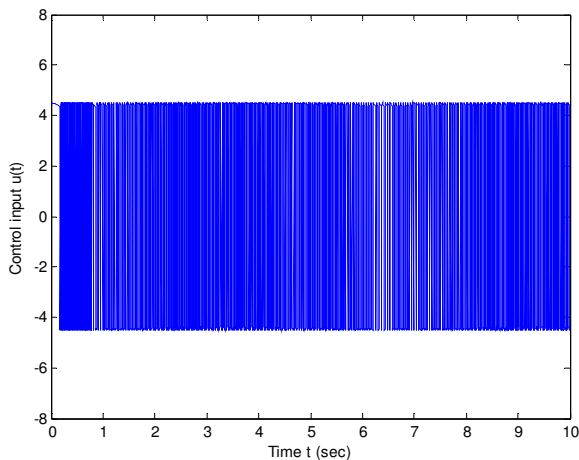


Fig. 2.4. Control input $u(t)$

Remark 3.1: Figure 3 shows the convergence comparison between the terminal sliding variable $s(t)$ and $\dot{e}(t)$, the first-order derivative of the tracking error $e(t)$, where two parameters u_n and u_p in (17) and (20) are chosen as:

$$u_n = -2.8, \text{ and } u_p = 2.8 \quad (30)$$

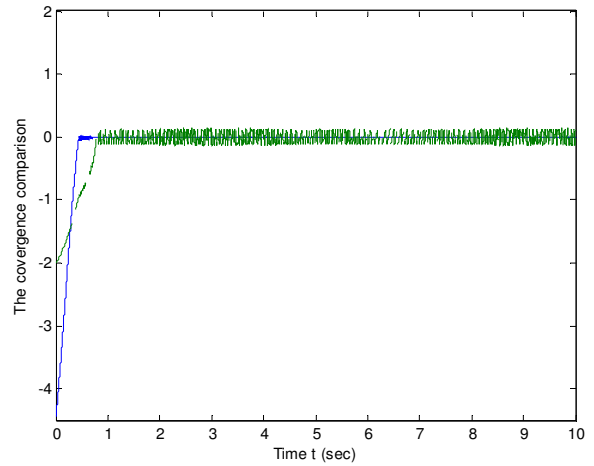


Fig. 3. Convergence comparison between $s(t)$ and $\dot{e}(t)$

It is seen that, before the terminal sliding variable $s(t)$ converges to zero, $|\dot{e}(t)| > 1$, which makes that $\rho(t)$ in (12) satisfies

$$\rho(t) \geq \delta > 0 \quad (31)$$

(31) guarantees that the variable structure control signal $u(t)$ in (18) and (21) can drive the terminal sliding variable $s(t)$ to converges to zero in a finite time, and then the tracking error converges to zero in a finite time on the terminal sliding mode surface $s(t) = 0$.

III. CONCLUSION

A new terminal sliding mode tracking control scheme has been developed in this paper for a class of nonminimum phase systems with uncertain dynamics. The main contribution of this research is that, unlike the design of conventional sliding mode control systems, Lyapunov stability theory in this paper is used to determine the bounded information of control signal and its first-order derivative, a dynamic control law can then be designed, subject to the bounded conditions, to drive the terminal sliding variable to zero in a finite time, and the tracking error can then converge to zero in a finite time on the terminal sliding mode surface. A simulation example has been given in support of the proposed new terminal sliding mode control scheme. The extension of this scheme to the terminal sliding mode control of high-order nonminimum phase systems with more than 2 zeros on the right half s -plane is under authors' investigation.

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